EFFECT OF HALL CURRENTS ON THE FLOW OF A CONDUCTING GAS AT HIGH FLOW VELOCITIES

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In [1] the flow of a compressible fluid was examined for the case when the conductivity $\sigma = \infty$ with account for the Hall effect. Oates [2] solves the problem of the influence of Hall currents on the flow in an accelerator for channels having a very small ratio of height to length when the velocity component in the direction of the channel height may be assumed to be zero. The problem of the influence of Hall currents on the flow of a conducting gas of finite conductivity is solved below for the case when the gas is accelerated to high velocities (~50–100 km/sec) with account for the presence of two velocity components.

§1. We shall consider the steady flow of a conducting gas in a channel of rectangular shape and constant cross section. We direct the x axis along the channel axis, the y axis in the direction of its height, and the z axis perpendicular to the side walls. Our basic assumptions are as follows.

1. The external magnetic field has one component directed along the z axis whose strength $H_z = H(x)$ may depend on x. Strictly speaking, for $H \neq$ const the component H_x will also be nonzero, equal in order of magnitude to $H_x \sim (z_0/L)\Delta H$, where z_0 is the channel width, L its length, and ΔH the characteristic magnitude of the variation in H. In what follows we shall assume that either $\Delta H \ll H$ or $z_0/L \ll 1$. In both cases $H_x \ll H$, so that the component of the magnetic field along the x axis may be neglected.

2. The magnetic Reynolds number $R_m \ll 1$; in comparison with the external field, the induced magnetic field may be neglected.

3. The effect of viscosity may be neglected

4. A potential difference which varies along the length of the channel $\varphi(x)$ is applied to its upper and lower walls, but is such that the component of the external electric field along the x axis is small in comparison with the component along the y axis.

5. The distortion of the electric field at the channel boundaries has little effect on the gas motion in the region of the channel under consideration.

6. The conductivity σ is constant in the flow region considered; the quantity $k = \omega\tau$ is also constant, where ω is the cyclotron frequency of electron gyration, and τ is the free time for an electron. We shall also assume that the quantity k is not very small (Hall currents are significant), but still $k < 1$, so that $1 + k^2 \approx 1$ (for example, $k \approx 0.1$—0.3).

7. The motion may be considered two-dimensional, i.e., the boundary conditions are such that all the required quantities are independent of the z coordinate.

8. The gas is perfect and obeys the Clapeyron equation of state.

9. Everywhere in what follows we shall consider acceleration in strong magnetic fields, when the characteristic velocity values attain magnitudes of the order $u \sim 5\cdot10^6/10^7$ cm/sec.

With these assumptions, one may neglect the pressure gradient in comparison with the momentum terms in the equations of motion.

In fact, assuming that in the process of motion all variables change by an order of magnitude and making an order-of-magnitude estimate of the terms in the component of motion along the x axis, we have

$$p_x' / \rho u u_x' \sim RT / u^2 .$$

Here p is the pressure, ρ the density, T the temperature of the gas, u the velocity component of the gas on the x axis, and R the gas constant.

For many gases and alkali metal vapors at velocities u such as we are considering, this ratio has a value of the order 0.001 and less, even for temperatures $T \sim 10^{4\circ}$ K.

An estimate of terms in the y component of the equation of motion gives

$$p_y' / \rho u v_x' \sim (RT / u^2)(L^2 / y_0^2) .$$

Here v is the y component of the gas velocity and y_0 the channel height. In deriving this relation it is assumed that $v/u \sim y_0/L$. For a channel length 5-10 times greater than its height, and the same flow parameters as above, the right-hand side equals 0.025-01.

In the case under consideration, Ohm's law has the form (see, for example, [3])

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{H} / c) - k(\mathbf{j} \times \mathbf{H}) / H .$$

Here j is the current density, E the electric field strength, H the magnetic field strength, V the flow velocity, and c the speed of light in a vacuum.

Solving for the current density components on the x and y axes and taking into account the assumptions made above regarding the electric and magnetic fields, we obtain

$$j_x = [\sigma / (1 + k^2)] [E_x + vH / c - k(E_y - uH / c)], \quad (1.1)$$

$$j_y = [\sigma / (1 + k^2)] [E_y - uH / c + k(E_x + vH / c)]. \quad (1.2)$$

The equation of continuity of electric current $\partial j_x/\partial x + \partial j_y/\partial y = 0$ then gives

$$E_y' + (H / c)[v_x' - u_y' + k(u_x' + v_y')] = 0. \quad (1.3)$$

Here $E_y = E$, and the primes and subscripts denote partial derivatives. In deriving (1.3) we took into account the fact that rot $E = 0$, and also that in view of the suppositions which have been made, $\partial E_x / \partial x \sim (y_0^2/L^2)\partial E/\partial y$ and that consequently $\partial E_x/\partial x$ may be discarded as a small quantity. With account for the estimates made, the equations of continuity and motion assume the form

$$(\rho u)_x' + (\rho v)_y' = 0, \qquad \rho u u_x' + \rho v u_y' = H j_y / c,$$
$$\rho u v_x' + \rho v v_y' = -H j_x / c. \qquad (1.4)$$

The system of equations (1.1)-(1.4) is a closed system of six equations in six unknown functions ρ, u, v, E, j_x and j_y.

We can take boundary conditions for the system in the following form: at a certain cross section $\tau = 0$ where the Hall currents do not yet exert any influence, all the parameters may be regarded as given:

$$u = u_{00} = \text{const}, \quad v = 0, \quad \rho = \rho_{00} = \text{const} \quad \text{at} \quad x = 0. \quad (1.5)$$

In addition, on the upper and lower walls, we have

$$v = 0 \quad \text{at } y = 0 \text{ and } y = y_0, \quad \int_0^{y_0} E dy = \varphi(x). \quad (1.6)$$

The last condition expresses the fact that the potential difference between the upper and lower channel walls is a given quantity.

System (1.1)-(1.4) may be solved, in the general case, by numerical methods only. Thus, we will seek an approximate solution, expanding all the required quantities in series in powers of k, and confining ourselves to terms linear in k. We set

$$u = u_0 + k u_1, \quad v = v_0 + k v_1,$$
$$\rho = \rho_0 + k \rho_1, \quad E = E_0 + k E_1. \qquad (1.7)$$

Without carrying out the calculations, we can make some general observations about the flow pattern in the case under consideration. In the absence of Hall currents (k = 0) the motion in the channel would be one-dimensional, given our assumptions. Hall currents lead to the appearance of a force, directed along the y axis and acting on all the gas particles. This force brings about motion in the direction of the y axis. Estimates made above show that in the case considered the particle velocity in the direction of the y axis will be supersonic.

Since we are neglecting viscous effects, the gas particles that acquire a velocity in the direction of the y axis break loose from the lower wall, where a region of vacuum develops. As a result of viscosity and diffusion effects, there will not be a complete vacuum under real conditions, but there will be a zone of extremely low density. We may thus assume that this zone will have finite conductivity.

Braking from supersonic velocity to $v = 0$ should occur at the upper wall of the channel. Thus a compression shock will arise at the upper wall. Therefore, the channel flow separates into three zones in the case under consideration: vacuum zone at the lower wall, a core flow in the channel, and a zone behind the compression shock at the upper wall.

If $y_0 - y^1$ is the height of the zone behind the compression shock, then one may write $y_0 - y^1 = y_0^1 + k y_1^1$, as in (1.7). However, in the absence of Hall currents $k = 0$, there is no shock, i.e., $y_0^1 = 0$. Such considerations are also valid for the height of the vacuum zone at the lower wall. We may thus conclude that the ratio of heights of the vacuum zone and the zone behing the compression shock to the channel height must be of the order of k.

2. We shall consider the flow in the core of the stream. Since u_0, v_0, ρ_0, E_0 are the values of the variables in the case when there are no Hall currents $k = 0$ and the motion in the channel is one-dimensional, $v_0 = 0$ and u_0, ρ_0 and E_0 can depend only on x. Noting this, we substitute expression (1.7) in (1.3), (1.4), having previously eliminated j_x and j_y, and also in the boundary conditions (1.5), (1.6). Equating terms with like powers of k, we obtain a system of equations for determining the magnitudes of the zero-th and first approximations

$$v_0 = 0, \qquad u_{0y}' = 0, \qquad E_{0y}' = 0$$

$$(\rho_0 u_0)_x' = 0, \qquad \rho_0 u_0 u_{0x}' = (\sigma H / c)(E_0 - u_0 H / c), \quad (2.1)$$

$$u_0 = u_{00}, \quad \rho_0 = \rho_{00} \quad \text{at} \quad x = 0, \qquad \int_0^{y_0} E_0 dy = \varphi(x), \quad (2.2)$$

$$(\rho_0 u_1 + \rho_1 u_0)_x' + (\rho_0 v_1)_y' = 0,$$
$$(\rho_0 u_1 + \rho_1 u_0) u_{0x}' + \rho_0 u_0 u_{1x}' = (\sigma H / c)(E_1 - H u_1 / c),$$
$$\rho_0 u_0 v_{1x}' = -(\sigma H / c)(H v_1 / c - E_0 + H u_0 / c),$$
$$E_{1y}' + (H / c)(v_{1x}' - u_{1y}' + u_{0x}') = 0, \qquad (2.3)$$

$$u_1 = 0, \quad v_1 = 0, \quad \rho_1 = 0 \quad \text{at } x = 0, \qquad \int_0^{y_0} E_1 dy = 0. \quad (2.4)$$

Here the integral of E_1 is taken over the whole height of the channel, and not over the height of the core only. This may be done with the degree of accuracy assumed, since the height difference between the core of the flow and the whole channel will be of the order of k, as was pointed out above. However, integral (2.4) is itself a coefficient in the term of order k in the series expansion of integral (1.6). Thus, changing the limits of integration leads to a correction of the order of k^2 only, i.e., to a correction of such an order as we everywhere neglect.

Integrating equations (2.1) and taking conditions (2.2) into account, we find

$$\rho_0 u_0 = \rho_{00} u_{00}, \qquad E_0 = \varphi / y_0,$$
$$u_0 = e^{-\Lambda(x)} \left\{ u_{00} + \frac{\sigma}{c \rho_{00} u_{00} y_0} \int_0^x H \varphi e^{\Lambda(x)} dx \right\},$$
$$\Lambda(x) = \frac{\sigma}{c^2 \rho_{00} u_{00}} \int_0^x H^2 dx. \qquad (2.5)$$

The system of equations (2.3) for functions of the first approximation is a system of four linear first order partial differential equations. Its solution may be reduced to the calculation of quadratures.

We substitute the expressions for u_0, ρ_0 and E_0 in the third equation of (2.3). This turns out to be a linear equation in the unknown function v_1, not containing the derivative v_{1y}'; the coefficients of this equation depend on x only. Integrating over x and taking $v_1 = 0$ for $x = 0$, we obtain

$$v_1 = e^{-\Lambda(x)} \left\{ \frac{\sigma}{c\rho_{00}u_{00}y_0} \int_0^x H\varphi e^{\Lambda(x)} dx - \frac{\sigma}{c^2\rho_{00}u_{00}} \int_0^x H^2 u_0 e^{\Lambda(x)} dx \right\}. \quad (2.6)$$

It is clear from expression (2.6) that v_1 is a function of one variable x only. Taking this into account, we find from the first equation of (2.3)

$$\rho_0 u_1 + \rho_1 u_0 = 0 \qquad \rho_1 = -\rho_0 u_1 / u_0 . \quad (2.7)$$

Boundary conditions (2.4) for u_1 and ρ_1 were used in deriving formulas (2.7). Eliminating ρ_1 from the first and second equations of (2.3), we obtain a system of two partial differential equations of the first order for determining the unknown functions u_1 and E_1

$$u_{1x}' + \frac{\sigma}{c^2\rho_{00}u_{00}} H^2 u_1 = \frac{\sigma}{c\rho_{00}u_{00}} H E_1,$$

$$E_{1y}' - \frac{H}{c} u_{1y}' = -\frac{1}{c} H (v_1 + u_0)_x' . \quad (2.8)$$

Here v_1, u_0 and H are known functions of x.

We differentiate the first equation with respect to y and substitute E_{1y}' from the second equation into the relation thus obtained

$$u_{1xy}'' = -\sigma H^2 (v_1 + u_0)_x' / c^2\rho_{00}u_{00} .$$

Integrating this equation twice with respect to x and y, and taking conditions (2.4) into account, we calculate u_1, after which we find E_1 from the first equation of (2.8)

$$u_1 = -\frac{\sigma}{c^2\rho_{00}u_{00}} y \int_0^x H^2 (v_1 + u_0)_x' dx + \Phi(x),$$

$$E_1 = \frac{H}{c} u_1 + \frac{c\rho_{00}u_{00}}{\sigma H} u_{1x}' . \quad (2.9)$$

Here $\Phi(x)$ is an arbitrary function of x with $\Phi(0) = 0$, since $u_1 = 0$ for $x = 0$. The function $\Phi(x)$ is determined from the last equation of (2.4). We set u_1 in the expression for E_1 and integrate over y from zero to $y = y_0$. Since the left-hand side vanishes, we thus obtain an ordinary linear differential equation in $\Phi(x)$

$$\Phi_x' + \frac{\sigma H^2}{c^2\rho_{00}u_{00}} \Phi =$$

$$\frac{\sigma y_0 H}{2c^2\rho_{00}u_{00}} \left\{ \frac{\sigma H}{c^2\rho_{00}u_{00}} \int_0^x H^2 (v_1 + u_0)_x' dx + H (v_1 + u_0)_x' \right\},$$

whence, considering that $\Phi(0) = 0$,

$$\Phi = \frac{\sigma y_0}{2c^2\rho_{00}u_{00}} e^{-\Lambda(x)} \int_0^x \left\{ \frac{\sigma H^2}{c^2\rho_{00}u_{00}} \int_0^x H^2 (v_1 + u_0)_x' dx + \right.$$

$$\left. + H^2 (v_1 + u_0)_x' \right\} e^{\Lambda(x)} dx . \quad (2.10)$$

Formulas (2.5)–(2.10) give a solution of the problem posed for an arbitrary dependence on x of the magnetic field H and the potential difference φ on the electrodes. If H = const and φ = const, the integrals in formulas (2.5)–(2.10) may be evaluated.

Without going through the calculations, we cite the formulas for this case

$$u_0 / u_{00} = 1 + U (1 - e^{-x^*}) + kU (0.5y_0^* - y^*) \times$$

$$\times (x^* e^{-x^*} - e^{-x^*} + 1), \quad v / u_{00} = kUx^* e^{-x^*},$$

$$Ec / Hu_{00} = 1 + U + kU (0.5y_0^* - y^*) (e^{-x^*} + 1),$$

$$\lambda = \sigma H^2 L / c^2\rho_{00}u_{00}, \quad U = -1 + \varphi c / y_0 H u_{00},$$

$$x^* = \lambda x / L, \quad y^* = \lambda y / L, \quad y_0^* = \lambda y_0 / L . \quad (2.11)$$

Formulas (2.11) show that when Hall currents are present the component of velocity on the x axis u is greater in the lower half of the channel $y < 0.5 y_0$, and less in the upper half than when Hall currents are absent (for k = 0). Increase of velocity u leads to the appearance of a stronger electric field E in the lower half of the channel. The velocity component on the y axis in the presence of Hall currents turns out to be constant over the channel cross section in the first approximation. At first, it increases along the length of the channel, reaching a maximum at the cross section $x/L = 1/\lambda$, and then decreases. The quantity max $v = ku_{00}Ue^{-1}$ does not depend on λ. The function $v(x/L)$ is shown in Fig. 1 for different values of λ; here $V^* = v/ku_{00}U$. It is clear from (2.11) and Fig 1 that for $\lambda \geq 1$, the cross section where v reaches a maximum lies inside the channel, and for $\lambda < 1$, outside it. The larger λ, the more rapidly the value of v changes. In order to explain this behavior of v we shall consider the electric current flow in the channel. Rejecting small higher-order terms, we have from formulas (1.1) and (1.2)

$$j_y = \sigma (E - uH / c), \quad j_x = \sigma [vH / c - k (E - uH / c)] . \quad (2.12)$$

Substituting the values for E, u, and v from (2.11) in these formulas, and retaining only the lowest-order terms we obtain

$$j_y = (\sigma H u_{00} / c) U e^{-x^*},$$

$$j_x = (\sigma H u_{00} / c) kU (x^* - 1) e^{-x^*} . \quad (2.13)$$

Hence we have the differential equation for the electric current lines and the lines of electric current themselves (X^* is the value of x^* for $y^* = 0$):

$$dx^* / dy^* = j_x / j_y = k (x^* - 1),$$

$$| x^* - 1 | = | X^* - 1 | e^{ky^*} . \quad (2.14)$$

The pattern of electric current lines calculated from formula (2.14) for $\lambda = 4$, $k = 0.2$ and $y_0/L = 0.3$ is shown in Fig. 2. It follows from formula (2.13) and Fig. 2 that j_x is positive for $x^* > 1$ and negative for $x^* < 1$. Since $x^* = \lambda x/L$, the cross section where j_x changes sign lies inside the channel for $\lambda > 1$. This is connected with the fact that at first $v = 0$ and $j_x < 0$, as is clear from (2.12). For large λ (i.e., in the case of strong magnetic fields) the velocity along the axis u rapidly attains values close to max u. $E - uH/c$ then becomes a small quantity, less than vH/c. It is clear from formula (2.12) that in this case $j_x > 0$. Change of direction of the electric current component on the x axis leads to a change of direction of the force acting along the y axis. Thus, beginning from the cross section $x^* = 1$, v, the component of velocity on the y axis, will decrease. $x^* < 1$ everywhere in the channel for $\lambda < 1$. Thus, for $\lambda < 1$ the velocity v increases monotonically along the whole channel.

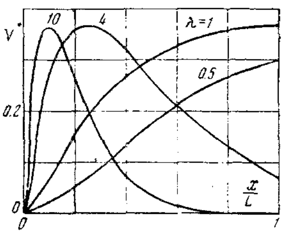

Fig. 1

With an accuracy to terms of order k the differential equation for current lines in the flow core has the form

$$dy^*/dx^* = v/u = kUx^* \left[(1+U)e^{x^*} - U\right]^{-1}.$$

Hence, on integrating, we have

$$y^* = Y^* + kU \int_0^{x^*} \frac{x^*}{(1+U)e^{x^*} - U} dx^* \qquad (2.15)$$

where Y^* is the value of y^* on the initial cross section for the current line under consideration. It follows from (2.15) that in the first approximation the current lines may be obtained one from the other by a shift along the y axis.

Figure 3 shows the form of the current lines in the flow core calculated from formula (2.15). The solid curve gives the form of the current lines for very large values of U (this corresponds to small values of u_{00} in comparison with the maximum velocity). The current line for U = 1 is shown by the broken curve.

3. We shall now consider the flow behind the compression shock which forms at the upper wall of the channel. Since the core flow is known, the flow problem close to the upper channel wall is similar in the case under consideration to the problem of highly supersonic flow past a body, and may be solved by the methods used in this case [4].

Two circumstances allow the solution of the problem to be simplified considerably. First of all, estimating the magnitude of v, the velocity component on the y axis behind the shock, we obtain

$$v/u \sim (y_0 - y^1)/L \sim ky_0/L. \qquad (3.1)$$

Here y^1 is the coordinate of the shock. Since the magnitude of the ratio $y_0/L \sim 0.1 - 0.3$, i.e., of the order k, it follows from (3.1) that $v/u \sim k^2$ behind the shock, and we may assume that behind the shock $v \approx 0$ with the assumed degree of accuracy.

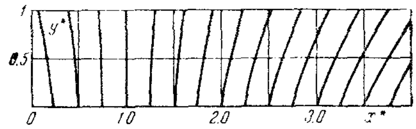

Fig. 2

Secondly, as was pointed out above, RT/v^2 may be of the order 0.025–0.1 in the flow core. We will therefore examine below in detail the case when the velocity component along the normal to the shock u_n is much greater than the velocity of sound. We may then assume approximately [4]

$$\rho^1/\rho^\circ = (\varkappa+1)/(\varkappa-1), \quad u_n^1/u_n^\circ = (\varkappa-1)/(\varkappa+1). \qquad (3.2)$$

Here $\varkappa$ is the ratio of specific heats, ρ^1 the density behind the shock, ρ° the density in the flow core in front of the shock, u_n^1 the normal velocity component behind the shock and u_n° in front of the shock. If the gas is totally ionized, then it may be assumed that $\varkappa$ does not change on passing through the shock.

u_τ° designates the velocity component in the flow core tangential to the shock, u_τ^1 the tangential velocity component behind the shock, α the angle of inclination of the shock to the upper channel wall (Fig.4). Then the equations

$$u^1 = u_\tau^1 \cos\alpha + u_n^1 \sin\alpha, \quad v^1 = -u_\tau^1 \sin\alpha + u_n^1 \cos\alpha \qquad (3.3)$$

$$u_\tau = u\cos\alpha - v\sin\alpha, \qquad u_n = u\sin\alpha + v\cos\alpha \qquad (3.4)$$

are valid.

Since in our case $v^1 = 0$, it follows from the second equation of (3.3) that

$$\mathrm{tg}\,\alpha = u_n^1/u_\tau^1 = u_n^1/u_\tau^\circ = [(\varkappa-1)/(\varkappa+1)]\, u_n^\circ/u_\tau^\circ. \qquad (3.5)$$

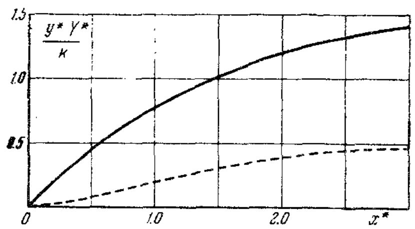

Fig. 3

Here we have used the relation (3.2) and the fact that the tangential velocity component does not chang its value on passing through the shock, i.e., $u_\tau^1 = u_\tau^\circ$. Substituting the expressions u_n° and u_τ° from (3.4) in (3.5), we obtain an equation for determining tg α:

$$v^\circ \mathrm{tg}^2\alpha - 2(u^\circ \mathrm{tg}\,\alpha)/(\varkappa+1) + (\varkappa-1)v^\circ/(\varkappa+1) = 0$$

where v° and u° are the velocity components on the y and x axes in front of the shock. Hence

$$\operatorname{tg}\alpha = u^\circ / v^\circ (\varkappa + 1) \pm$$
$$\pm [u^{\circ 2} / v^{\circ 2} (\varkappa + 1)^2 - (\varkappa - 1) / (\varkappa + 1)]^{1/2}. \quad (3.6)$$

Since $u^\circ / v^\circ \sim 1/k$ is a very large quantity, the plus sign must correspond to an almost normal shock almost perpendicular to the upper wall of the channel. This has no physical meaning in relation to the problem of plasma acceleration by strong magnetic fields. Thus, the minus sign must be taken in equation (3.6). Confining ourselves to terms of order k, we find from (3.6) and (3.4)

$$\operatorname{tg}\alpha = (\varkappa - 1) v^\circ / 2u^\circ, \qquad u^1 = u_\tau^1 = u_\tau^\circ = u^\circ. \quad (3.7)$$

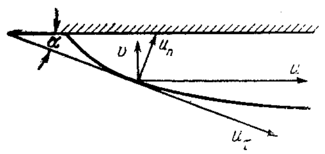

Fig. 4

Equations (3.2) and (3.7) allow us to determine the plasma parameters behind the shock from known values of the parameters in front of the shock, and also the angle of inclination of the shock. The current lines behind the shock are, with the degree of accuracy assumed, straight and parallel to the channel wall (since $v^1 = 0$). Thus behind the shock the equations of continuity and motion assume the form

$$(\rho u)_x' = 0, \qquad \rho u u_x' = H j_y / c. \quad (3.8)$$

Integrating the first equation and taking (3.2) and (3.7) into account, we obtain $\rho u = \rho^1 u^1 = [(\kappa + 1)/(\kappa - 1)]\rho^\circ u^\circ$. In this equation ρ° and u° are the values of density and velocity in the flow core in front of the shock at the point where the current line in question intersects the shock. Expanding ρ° and u° in series in powers of k and considering equations (2.5) and (2.7), we obtain, with an accuracy to terms of order k^2,

$$\rho^\circ u^\circ = \rho_{00} u_{00}, \qquad \rho u = [(\varkappa + 1) / (\varkappa - 1)] \rho_{00} u_{00}. \quad (3.9)$$

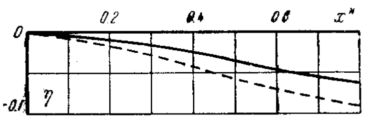

Fig. 5

We now note that the current j_y in the second equation of (3.8) is a quantity known correct to terms of order k^2, which can be calculated from the flow parameters in the core. Actually, from the continuity equation for electric current we have for j_y behind the shock

$$j_y = j_y^1 - \int_{y^1}^{y} \frac{\partial j_x}{\partial x} dy, \quad (3.10)$$

where j_y^1 is the value of j_y for $y = y^1$.

However, it follows from (1.1) that $\partial j_x / \partial x$ is of the order of k, and since the inteval of integration behind the shock is also of order k, the integral on the right-hand side of (3.10) will be of order k^2, and we may take $j_y = j_y^1$ with the assumed degree of accuracy. Considering this and also (3.9), we find the flow velocity in the zone behind the shock by integrating the second equation of (3.8)

$$u = u^1 + \frac{\varkappa - 1}{(\varkappa + 1) c \rho_{00} u_{00}} \int_{x^1}^{x} H j_y dx. \quad (3.11)$$

Formulas (3.7), (3.9), and (3.11) give the distribution of flow parameters behind the shock and the angle of inclination of the shock for arbitrary dependence on x of the magnetic field strength H and potential difference φ. In the case when H and φ are constant, we easily obtain the form of the shock from the first equation of (3.7), since $dy^1/dx = -\operatorname{tg}\alpha$. Setting u° and v° from (2.11) in (3.7), discarding terms of order k^2 and integrating over x, we obtain

$$y^{1*} = y^{0*} - k \frac{\varkappa - 1}{2} U \int_0^{x^*} \frac{x^* dx^*}{(1 + U) e^{x^*} - U},$$
$$x^* = \frac{\lambda x}{L}, \qquad y^* = \frac{\lambda y}{L}. \quad (3.12)$$

The form of the shock, calculated from (3.12), is shown in Fig. 5, where

$$\eta = \frac{2}{(\varkappa - 1) k} (y^{1*} - y_0^*).$$

The continuous curve is for U = 3, the broken curve for U = 8.

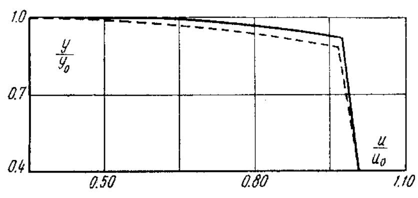

Fig. 6

Setting the values for E and u from (2.11) in (1.2), and discarding terms of order k^2, we find for the current density j_y behind the shock

$$j_y = (\sigma H u_{00} / c) U e^{-x^*} [1 - 0.5 k y^* (2 - x^*)].$$

Using this expression for j_y, we now determine the velocity from **(3.11)**

$$u(x^*, y^*)/u_{00} = 1 + U(1 - e^{-x^*}) -$$
$$- [2/(\varkappa + 1)]\, U\, [e^{-x^{1*}(y)} - e^{-x^*}] + 0.5kU \times$$
$$\times y^* \{-1 + (1 - x^*)\, e^{-x^*} -$$
$$- [2/(\varkappa + 1)]\, [(1 - x^*)\, e^{-x^*} + (1 - x^{1*})\, e^{-x^{1*}(y)}]\},$$

where $x^{1*}(y)$ is the value of x at which the current line y = const intersects the shock. This value of $x^*(y)$ is calculated from formula (3.12). With the help of (2.11) the formula for the velocity behind the shock may be transformed to

$$u(x, y) = u[x, y^1(x)] - [2/(\varkappa + 1)] \times$$
$$\times \{u[x, y^1(x)] - u[x^1(y), y]\} \qquad (3.13)$$

Hence the velocity on any current line behind the shock is less than the velocity in the flow core in front of the shock, by an amount equal to the difference between the velocities in the core in front of the shock in the channel cross section under consideration and in the cross section where the current line intersects the shock, multiplied by $2/(\kappa + 1)$.

Figure 6 shows the profile of u, the velocity component on the x axis, calculated for the core flow from formula (2.11) and behind the shock from formula (3.13). The curves correspond to values of the parameters $x^* = 1$, $k = 0.2$, $y_0^* = 0.3$, $\kappa = {}^5/_3$, U = 3 (continuous line) and U = 8 (broken line). The ratio of u, the velocity at a given point, to u_0, the velocity on the channel axis, is marked off on the abscissa axis, and on the ordinate axis the ratio of the y coordinate to the channel width y_0. The point of discontinuity of the curves corresponds to the coordinate of the shock in the channel cross section under consideration $x^* = 1$.

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